

Quark Model: masses

1) Observation of g.s. decuplet baryons

$$\Delta, \Sigma^*, \Xi^*, \Omega^-$$

which are $J=3/2$, and also completely symmetric
in flavor, lead to postulate of new QM, color

Hypothesis: quarks carry $SU(3)$ & color ~~singlets~~
hadrons bound color singlets - only color
singlets can reach large distances -
color confinement.

$$|\Omega\rangle = |\text{flavor}\rangle \otimes |\text{spin}\rangle \otimes |\text{color}\rangle$$

$$|\text{color}\rangle = \frac{1}{\sqrt{3}} \hat{\epsilon}_{abc} q^a q^b q^c$$

(singlets) (dominant color component)

for baryons, the flavor \otimes spin w.f. must be completely symmetric. For mesons, $\bar{q} \Gamma q$, simple sum over colors gives color neutral state ($\bar{q} = \frac{1}{\sqrt{3}}$) ($q = \frac{1}{\sqrt{3}}$), $\bar{q} q = \frac{1}{3}$

How far can we push this model?

We need to combine flavor & spin.

$$\langle B | H^* | B \rangle = \langle B | H^* | B \rangle + \langle B | E H^* | B \rangle$$

The decuplet states are easiest - totally symm in both, flavor & spin

$$\text{e.g. } |\Delta^{++}\rangle_{\frac{1}{2}^+} = |u u u\rangle_{\frac{1}{2}^+} \text{ from our definition}$$

$$|\Delta^+, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}} (|uud\rangle + |udu\rangle + |duu\rangle) - \frac{1}{\sqrt{3}} (|++-\rangle + |+-+\rangle + |-++\rangle)$$

How about octet baryons?

8 of $SU(3)$ is mixed symmetric.

$\frac{1}{2}$

recall, two mixed symmetric combinations

~~For mixing from 3x3 to 8x8~~

$$\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{3} \Rightarrow \begin{array}{c} \boxed{12} \\ \boxed{3} \end{array} \oplus \begin{array}{c} \boxed{13} \\ \boxed{2} \\ \boxed{4} \end{array}$$

and similarly with the $SU(2)$ spin 1/2 (2-d representation of $SU(2)$)

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} \Rightarrow \begin{array}{c} \boxed{12} \\ \boxed{2} \end{array} \oplus \begin{array}{c} \boxed{13} \\ \boxed{2} \\ \Psi_s^A \quad \Psi_s^S \end{array}$$

And so the total wavefunction for flavor \otimes spin can be formed as

$$|\mathcal{B}\rangle = |\psi_f^S\rangle \otimes |\psi_s^S\rangle + |\psi_f^A\rangle \otimes |\psi_s^A\rangle$$

$$\begin{aligned} \text{e.g. } |\psi^A\rangle &= \frac{1}{\sqrt{2}} (|ud\rangle - |du\rangle) |1u\rangle && \text{where order indicates} \\ &= \frac{1}{\sqrt{2}} (|udu\rangle - |duu\rangle) && \text{quark 1,2,3} \\ &&& \langle udu | uud \rangle = 0 \\ &&& \langle uud | uud \rangle = 1 \end{aligned}$$

$$\begin{aligned} |\psi^S\rangle &= \frac{1}{\sqrt{6}} [(|ud\rangle + |du\rangle) |1u\rangle - 2|uud\rangle] \\ &= \frac{1}{\sqrt{6}} (|udu\rangle + |duu\rangle - 2|uud\rangle) \end{aligned}$$

similarly, spin w.f.

$$|\psi_{1/2}^A\rangle = \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle)$$

$$|\psi_{1/2}^S\rangle = \frac{1}{\sqrt{6}} (|+-\rangle + |-+\rangle - 2|++\rangle)$$

What about the magnetic moments?

$$\mu_i = Q_i \left(\frac{e}{2m_i} \right) \quad Q_u = \frac{2}{3} \quad Q_d = -\frac{1}{3}$$

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}} \left(|\psi_f^s\rangle |\psi_s^s\rangle + |\psi_f^A\rangle |\psi_s^A\rangle \right)$$

$$|\text{K}\rangle = \frac{1}{\sqrt{18}} \left[\begin{array}{l} |uud\rangle (|+-+\rangle + |++-\rangle - 2|+-\rangle) \\ + |udd\rangle (|++-\rangle + |-+-\rangle - 2|+-\rangle) \\ + |duu\rangle (|+-+\rangle + |++-\rangle - 2|-+\rangle) \end{array} \right]$$

$$\mu_p = \sum_{q=1}^3 \langle p\uparrow | \mu_q (\sigma_3)_q | p\uparrow \rangle$$

$$(\sigma_3)_1 = \sigma_3 \otimes 1 \otimes 1$$

$$(\sigma_3)_2 = 1 \otimes \sigma_3 \otimes 1$$

$$(\sigma_3)_3 = 1 \otimes 1 \times \sigma_3$$

3 quarks

$$\Rightarrow \mu_p = \frac{1}{18} \left\{ \begin{array}{l} (\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d) + 4(\mu_u + \mu_u - \mu_d) \\ + (+\mu_u + \mu_d - \mu_u) + (-\mu_u + \mu_d + \mu_u) + 4(\mu_u - \mu_d + \mu_u) \\ + (+\mu_d - \mu_u + \mu_u) + (+\mu_d + \mu_u - \mu_u) + 4(-\mu_d + 2\mu_u) \end{array} \right\}$$

$$= \frac{3}{18} \left\{ 2\mu_d - 4\mu_d + 8\mu_u \right\}$$

$$\mu_p = \frac{1}{6} \left\{ 8\mu_u - 2\mu_d \right\} = \frac{1}{3} (4\mu_u - \mu_d)$$

What is $\mu_n = ?$

($d \leftrightarrow u$)

$$\mu_n = \frac{1}{3} (4\mu_d - \mu_u)$$

In isospin limit

$$\mu_u = -2\mu_d$$

$$\Rightarrow \frac{\mu_p}{\mu_n} = \frac{-8\mu_d - \mu_d}{4\mu_d + 2\mu_d} = -\frac{3}{2}$$

$$\frac{\mu_p^*}{\mu_n^*} = \frac{2.79}{-1.91} \approx 1.46 \quad \text{not too bad}$$

from our argument that $\mu \propto \frac{1}{m}$

$$\frac{e}{2m_p} \mu_p = \frac{1}{3} \{ 4\mu_u - \mu_d \} = \frac{1}{3} \{ 4\mu_u + \frac{1}{2}\mu_u \}$$

$$\mu_N = \frac{9}{2} \cdot \frac{1}{3} \mu_u = \frac{3}{2} \mu_u = \frac{3}{2} \cdot \frac{2}{3} \cdot \frac{e}{2m_u}$$

$$\mu_N / \mu_p = \frac{e}{2m_u}$$

$$\mu_p = \frac{m_p}{m_u} \approx \frac{940}{360} \approx 2.61 \quad (2.79) !$$

(90)

Thursday, 13th Feb

- page 81 $|f\text{lavor}\rangle \otimes |s\text{pin}\rangle$
- page 82
- page 88 $|p\uparrow\rangle, |u_p\rangle$
- page 89 $|u_n\rangle, |u_p/u_n\rangle$

How about $|N\rangle$?

i) construct flavor wf.

$$|A\rangle = \begin{array}{|c|c|} \hline u & s \\ \hline d & \\ \hline \end{array}$$

$$\text{we know } I_2|A\rangle = 0|A\rangle$$

$$I|A\rangle = 0|A\rangle$$

which tells us u and d must be in anti-symmetric combination.

$$\left. \begin{aligned} \text{recall } |\bar{n}^0\rangle &= \frac{1}{\sqrt{2}} (|\bar{u}u\rangle - |\bar{d}d\rangle) \\ \text{and sign flip for anti-quarks} \end{aligned} \right]$$

$$|N\rangle = \begin{array}{|c|c|} \hline u & s \\ \hline d & \\ \hline \end{array} = N_A \left\{ \begin{aligned} &(|uds\rangle - |dus\rangle) \\ &-(|sud\rangle - |sd\underline{u}\rangle) \\ &+ (|usd\rangle - |dsu\rangle) \end{aligned} \right\}$$

examining

by examination, requiring $u-d$ anti-symmetry
we deduce signs

OR - consider $|uds\rangle - |sud\rangle$

to get from 1st to 2nd, we first interchange $s-u$
(+) sign, then interchange ($u-d$) (-) sign

(e3)

What about $N_\pi = ?$

just recall $\langle uuds | duds \rangle = 0$

$$\begin{aligned} & \langle uuds | duds \rangle \\ &= \langle u|u\rangle \langle d|d\rangle \langle s|s\rangle \langle l|l\rangle \langle u|u\rangle \langle d|d\rangle \end{aligned}$$

$$\begin{aligned} &= \langle u|u\rangle \langle d|d\rangle \langle s|s\rangle \\ &= 0 \end{aligned}$$

$$N_\pi = \frac{1}{\sqrt{6}}$$

How about adding the spin wavefunctions?

We know u-d pair are anti-symmetric in flavor.
 \Rightarrow this means the corresponding spin w.f. must be anti-symmetric for u-d and also, to get a totally symmetric state. So we can similarly construct the spin w.f. and combine with flavor.

$$|\Lambda^\uparrow\rangle = N_{\Lambda^\uparrow} \left\{ \begin{array}{l} (\langle uuds \rangle - \langle duds \rangle)(\langle +-\rangle - \langle -+\rangle) \\ + (\langle suds \rangle - \langle sdus \rangle)(\langle ++\rangle - \langle +- \rangle) \\ + (\langle ldus \rangle - \langle usdl \rangle)(\langle -++\rangle - \langle ++-\rangle) \end{array} \right\}$$

Notice I have just cyclicly permuted the quarks
 $|uuds\rangle \rightarrow |suds\rangle \rightarrow |ldus\rangle$

Now we have 12 terms, instead of 6, so

$$N_{\Lambda^\uparrow} = \frac{1}{\sqrt{12}}$$

So what is $M_i = ?$

$$\langle \Lambda^{\uparrow} | \sum \mu_i | \Lambda^{\uparrow} \rangle$$

$$M_i = \frac{Q_i e}{2m_i}$$

$$= N_{\Lambda^{\uparrow}}^2 \left\{ \frac{Q_u}{m_p} (1) - \frac{Q_u}{m_n} (-1) - \frac{Q_u}{m_n} (+1) + \frac{Q_u}{m_n} \right\}$$

$$= \frac{1}{12} \left\{ M_u - M_d + M_s - M_u + M_d + M_s - \dots \right\}$$

$$= \mu_s = \frac{Q_s \cdot e}{2m_s}$$

$$= \frac{Q_s}{Q_u} \frac{m_u}{m_s} \cdot \frac{e}{2m_u}$$

$$= \frac{Q_s}{Q_u} \frac{m_u}{m_s} \cdot Q_u \cdot \frac{m_u}{m_p} \frac{e}{2m_p}$$

$$= Q_s \cdot \frac{m_u}{m_s} \cdot \langle p^{\uparrow} | \hat{M}_i | p^{\uparrow} \rangle$$

$$= -\frac{1}{3} \cdot \frac{m_u}{m_s} \cdot \mu_p \mu_N$$

$$= -\frac{1}{8} \cdot \frac{360}{536} \cdot (\mu_p \mu_N)$$

$$= -\frac{1}{3} \frac{m_u}{m_s} \cdot \frac{m_p}{m_n} \cdot \mu_N$$

$$= -\frac{1}{3} \cdot \frac{940}{836} \cdot \mu_N$$

$$\approx -0.6 \quad \mu_N$$

$$\text{exp } -0.613(4)$$

Helicity & Handedness

Helicity measures the component of angular momentum along the direction of motion

For a photon, the helicity eigenvalues are

$$\pm 1$$

for circularly polarized photons. Photons can

not have $J_z = 0$ (for photon traveling in z -direction)

because they are massless

- We can use the good-old-right-hand-rule to

determine if a particle has $> 0 \text{ or } < 0$ helicity

How does this work for spinors?

For Dirac fermions, the general Lorentz

transformation

$$\Lambda_{\frac{1}{2}}(w^{\mu\nu}) = e^{\frac{i}{2} w_{\mu\nu} S^{\mu\nu}}$$

General Lorentz

$$\Lambda(w^{\mu\nu}) = e^{\frac{i}{2} w_{\mu\nu} J^{\mu\nu}}$$

$$S^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$$

Let us examine Dirac equation using
chiral (Weyl) representation

$$\gamma_x^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_x^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_x^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma^{0k} = \frac{i}{4} [\gamma_0, \gamma_k] = \frac{i}{2} \begin{pmatrix} -\sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -\sigma_k \begin{pmatrix} 0 & 0 \\ 0 & \sigma_k \end{pmatrix}$$

$$\gamma^{kl} = \frac{i}{4} [\gamma^k, \gamma^l] = \frac{i}{2} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \cancel{\epsilon^{ijk}} \begin{pmatrix} \sigma_m & 0 \\ 0 & \sigma_m \end{pmatrix}$$

$$= \frac{i}{4} \left[\begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^l \\ -\sigma^l & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^l \\ -\sigma^l & 0 \end{pmatrix} \right]$$

$$= \frac{i}{4} (-) \begin{pmatrix} +[\sigma^k, \sigma^l] & 0 \\ 0 & [\sigma^k, \sigma^l] \end{pmatrix}$$

$$= \frac{-i}{4} 2i \epsilon^{klm} \begin{pmatrix} \sigma_m & 0 \\ 0 & \sigma_m \end{pmatrix}$$

$$= \frac{1}{2} \epsilon^{klm} \begin{pmatrix} \sigma_m & 0 \\ 0 & \sigma_m \end{pmatrix}$$

So the Lorentz transformation is block diagonal in
this representation

$$i\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

These represent prime and anti monopole
extremes

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$i\gamma^1\gamma^2\gamma^3\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$i\gamma^1\gamma^2\gamma^3\gamma^k = \begin{pmatrix} -\sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

map of \mathbb{R}^4 to (\mathbb{H}^2)

What is massless dirac eq.

$$i\gamma^1\gamma^2\gamma^3 [\gamma^\mu \partial_\mu] \psi = 0$$

$$= [i\gamma^1\gamma^2\gamma^3 \gamma^0 - i\gamma^1\gamma^2\gamma^3 \gamma^k \cdot \nabla^k] \psi = 0$$

$$\begin{bmatrix} i(\partial_0 + \sigma_k \nabla^k) & 0 \\ 0 & i(\partial_0 + \sigma_k \nabla^k) \end{bmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = 0$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\psi_R = \frac{1 + \gamma^5}{2} \psi = P_R \psi \quad P_R P_R = P_R$$

$$\psi_L = \frac{1 - \gamma^5}{2} \psi = P_L \psi \quad P_L P_L = P_L$$

$$P_R P_L = 0$$

$$P_L + P_R = 1$$

A massless Dirac fermion is really two distinct fermions, Left & Right handed.

For massless fermions - helicity = handedness

You can't boost past a massless particle to flip its helicity

as many fermions are free

The Massive Dirac eq.

$$i\gamma^1\gamma^2\gamma^3 \left(\partial^\mu - m \Gamma_{\mu} \right) \psi = 0$$

so there's only one equation of motion

$$= \begin{bmatrix} i(-\partial_0 + \vec{\sigma}^k \vec{\gamma}^k) & m \\ -m & i(\partial_0 + \vec{\sigma}^k \vec{\gamma}^k) \end{bmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

The mass term explicitly mixes the left & Right

handed fields. This has very important consequences
for our understanding of the SM.

$$B> \rightarrow (\bar{e}^+ e^-) = \bar{e}^+ \gamma^0 e^- \quad (0^+ - 1^-)$$

$$W> \rightarrow (\bar{e}^+ e^-) + (\bar{\nu}_e \nu_e) \quad H^0$$

$$H> \rightarrow (\bar{e}^+ e^-) = \bar{e}^+ \gamma^0 e^- \quad (0^+ - 1^-)$$

Matter fields

Consider self-energy of Electron

$$H^0 D^0 \rightarrow \bar{D}^0 D^0$$

$$H^0 D^0 \rightarrow \bar{D}^0 D^0$$

$$\text{Symmetry } D^0 - (\frac{1}{2})^+ D^0: (\frac{1}{2})^+ D^0 = (\frac{1}{2})^+$$

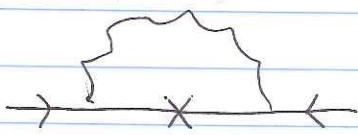
Massive fermion loop for helicity flip
↑ denotes helicity

The electro-magnetic interactions conserve handedness/helicity

$$F \in C_{\mu} \rightarrow \partial_{\mu} + i q A_{\mu} \quad \text{no spin-flip}$$

So the photon can not change the helicity

But the mass operator does flip helicity



This gives us a very powerful explanation as to

why the fermion masses are so light.

There is an underlying symmetry which protects

the mass. If you use a regularization

scheme which respects "chiral" symmetry, you

find the fermion masses are "multiplicatively"

normalized.

Examine Dirac spinor in momentum space, for

$$\psi(x) = u(p) e^{-ip^0 x} \quad p^0 > 0 \text{ solution}$$

$\Rightarrow \vec{p} = \vec{k}$ prof momentum imp. energy

not important

$$(i\gamma^\mu \partial^\mu - m)\psi \Rightarrow (\not{p}_\mu \gamma^\mu - m)u(p) = 0.$$

in the rest frame $p^\mu = (m, \vec{0})$

$$\Rightarrow (m\gamma^0 - m)u(p) = 0$$

$$m \begin{pmatrix} -m & u_1 \\ u_0 & -u_0 \end{pmatrix} u(p) = 0$$

$$u(p) = \sqrt{m} \begin{pmatrix} \xi \\ \bar{\xi} \end{pmatrix}, \quad \bar{\xi} + \xi = 1$$

later

$$\bar{\xi}_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \bar{\xi}_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

convenience

We can boost to another frame.

If we want particle to have rapidity γ ,

we boost the observer by $-\gamma$ (Passive boost = boost frame)
 (in z-direction) (active boost = boost particle)

$$\Lambda_{12}(-\gamma) u(m) = e^{-i\gamma S_0^3} u(m)$$

$$= \exp \left[-\frac{1}{2}\gamma \begin{pmatrix} 0^3 & 0 \\ 0 & -0^3 \end{pmatrix} \right] \sqrt{m} \begin{pmatrix} \xi \\ \bar{\xi} \end{pmatrix}$$

aff constant für Partikel

für reelle Energie prop für e- dosimetric Impulsfaktor α

$$\Lambda_{12}(-y) u(p) = \left[\cosh\left(\frac{y}{2}\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \sinh\left(\frac{y}{2}\right) \begin{pmatrix} \delta_3 & 0 \\ 0 & -\delta_3 \end{pmatrix} \right] \sqrt{m} \begin{pmatrix} z \\ \xi \end{pmatrix}$$

$$= \begin{bmatrix} e^{\frac{y}{2}} \left(\frac{1-\delta_3}{2} \right) + \tilde{e}^{-\frac{y}{2}} \left(\frac{1+\delta_3}{2} \right) & 0 \\ 0 & e^{\frac{y}{2}} \left(\frac{1+\delta_3}{2} \right) + \tilde{e}^{-\frac{y}{2}} \left(\frac{1-\delta_3}{2} \right) \end{bmatrix} \sqrt{m} \begin{pmatrix} z \\ \xi \end{pmatrix}$$

$\rightarrow -E(E-p)$

$$u(p) = \begin{pmatrix} \sqrt{E+p_3} \left(\frac{1-\delta_3}{2} \right) + \sqrt{E-p_3} \left(\frac{1+\delta_3}{2} \right) \\ \sqrt{E+p_3} \left(\frac{1+\delta_3}{2} \right) + \sqrt{E-p_3} \left(\frac{1-\delta_3}{2} \right) \end{pmatrix} z$$

$$E = m \cosh y \quad p_3 = m \sinh y$$

$$E \pm p_3 = m e^{\pm y}$$

$$(E) u^\dagger u = 2 E z + \xi = 2 E \cos \text{ang of 200%}$$

$$\bar{u} u = u^\dagger \gamma^0 u = 2 m z + \xi = 2 m$$

$$u_\pm = \sqrt{m} \begin{pmatrix} z_\pm \\ \xi_\pm \end{pmatrix}$$

z^\pm = Impuls in perpendic.

$$u_\pm \propto \begin{pmatrix} z_\pm \\ \xi_\pm \end{pmatrix} \quad , \quad z_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad z_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$u_+(\vec{p}) = \begin{pmatrix} \sqrt{E - p_3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E + p_3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \xrightarrow{E \gg m} \begin{pmatrix} 0 \\ \sqrt{2E} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$u_-(\vec{p}) = \begin{pmatrix} \sqrt{E + p_3} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E - p_3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \xrightarrow{E \gg m} \begin{pmatrix} \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 0 \end{pmatrix}$$

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So, the Weyl basis is convenient for extreme relativistic fermions

We can define the helicity operator

$$\hat{h} = \vec{p} \cdot \vec{\sigma} = \frac{1}{2} p^k \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

$$P_{h\pm}(z) = \frac{1 \pm 2h}{2} = \frac{1}{2} \begin{pmatrix} 1 \pm \sigma_3 & 0 \\ 0 & 1 \pm \sigma_3 \end{pmatrix}$$

u_+ describes positive helicity particle moving in

+z-direction

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu - m) \psi = \bar{\psi}_L i \gamma^\mu \psi_L + \bar{\psi}_R i \gamma^\mu \psi_R - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

For massless fermions, the chiral and helicity projectors commute, so for massless fermions, they are the same operation